

Inference on a new lifetime distribution for a parallel-series system under progressively type-II censored samples

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Abstract. In this paper we introduce a new lifetime distribution with increasing, decreasing or upside-down bathtub shaped hazard rates, called doubly Poisson exponential distribution. One of the motivations of the new distribution is that it may represent the lifetime of units connected in a parallel-series system. Several properties of the new distribution are discussed. Based on progressive type-II censoring, six estimation methods for the involved parameters are considered. The methods are maximum likelihood, moments, least squares, weighted least squares and Bayes (using linear-exponential and general entropy loss functions) estimations. Bayes estimates for the parameters are obtained using Markov chain Monte Carlo algorithm. The performance of these methods is compared through an extensive numerical simulation, based on mean of mean squared errors and mean of relative absolute biases. Two real data sets are used to compare the new distribution with other five distributions. The comparison shows that the former distribution is better to fit the data than the other five distributions.

Keywords: parallel-series system, progressive type-II censoring, exponential distribution, maximum likelihood, moments, least squares, weighted least squares and Bayes estimations, simulation.

Bayesian estimation on the exponentiated Pareto distribution under type II censoring

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Abstract. In this paper, Bayes estimates of the two shape parameters, reliability and failure rate functions of the exponentiated Pareto lifetime model are derived from complete and type II censored samples. When the Bayesian approach is concerned, conjugate priors for either the one or the two shape parameters cases are considered. An approximation form due to Lindley (1980) is used for obtaining the Bayes estimates under the squared error loss and LINEX loss functions. The root-mean square errors of the estimates are computed. Comparisons are made between the Bayes estimators.

Keywords: Bayes estimators, exponentiated Pareto distribution, LINEX loss function, squared error loss function.

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Pointwise estimation of bivariate Pickands dependence function: A Bernstein copula approach

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Abstract. Modeling dependence structures of joint extreme events has been interest of various applicational areas, such as, environmental science, insurance, finance, etc. Pickands dependence function that characterizes the extreme-value copula is widely used to model these extreme events. In this study, Bernstein copula approximation is used to estimate Pickands dependence function. Pointwise estimation procedure with a visual tool is proposed for investigating the extreme-value dependence structure. The performance of the estimator is presented with a simulation study. Test results mainly show that the estimator has a good performance in detecting the tail behavior.

Keywords: extreme-value, Bernstein copula, Pickands dependence function.

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Joint signatures

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Abstract. In this talk, I will introduce the notion of joint signatures of two systems and present some of their properties including mixture representations for joint distributions of lifetimes of the two systems. I will then use this representation to develop some statistical inferential methods for characteristics of both systems and components based on system lifetime data. I will present some examples to illustrate the results developed. Finally, I will conclude the talk by mentioning some further issues that are worth of further study.

Asymptotic behavior of the joint record values, with applications

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Abstract. The class of limit distribution functions of the joint upper record values, as well as the joint of lower record values, is fully characterized. Sufficient conditions for the weak convergence are obtained. As an application of this result, the sufficient conditions for the weak convergence of the record quasi-range, record quasi-midrange, record extremal quasi-quotient and record extremal quasi-product are obtained. Moreover, the classes of the non-degenerate limit distribution functions of these statistics are derived.

Keywords: weak convergence, record values, joint record values, record functions.

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1. Barakat H.M., Abd Elgawad M.A. Asymptotic behavior of the joint record values, with applications. (submitted)

Limit distributions of generalized order statistics in a stationary Gaussian sequence

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Abstract. In this paper we study the limit distributions of extreme, intermediate and central m -generalized order statistics (gos), as well as m -dual generalized order statistics (dgos), of a stationary Gaussian sequence under equi-correlated set up. Moreover, the result of extremes is extended to a wide subclass of gos, as well as dgos, (which contains the most important models of ordered random variables), when the parameters $\gamma_{1,n}, \gamma_{2,n}, \dots, \gamma_{n,n}$ are assumed to be pairwise different.

Keywords: Gaussian sequences, generalized order statistics, dual generalized order statistics.

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1. Barakat H.M., Nigm E.M., Abo Zaid E.O. Asymptotic behavior of the joint record values, with applications. (submitted)

The estimations under power normalization for the tail index, with comparison

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Abstract. The objective of this research is to suggest two classes of moment and moment ratio estimators under power normalization for the tail index. Moreover, for quantitative comparison of the suggested estimators and other estimators, we use a mean square error criterion. The problem of weighting between the linear and power models to describe the given extreme data is challenging. For this purpose, we suggest the coefficient variation criterion. A simulation study is conducted, to assess and compare the accuracy of the suggested estimators and other estimators, as well as the suggested statistical criterions. The suggested estimators and other estimators, as well as the suggested criterions are used to analyze a real data sets. All computations in this work are performed by R-package.

Keywords: power normalization, generalized Pareto distributions, Hill estimators, moment estimator, moment ratio estimator.

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1. Barakat H.M., Nigm E.M., Khaled O.M., Alaswedz H.A. The estimations under power normalization for the tail index, with comparison. (submitted)

On some new models of multivariate record values

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Abstract. The theory of record values in multivariate sequences of random variables is considered. One of the considered models is based on coordinatewise ordering of multivariate observations. The distributional properties of record values in a new scheme of multivariate records is presented. Some examples with well known bivariate distributions as underlying distributions of original sample are given and graphical illustrations are provided. In the second record model, we consider the N-ordering scheme for random vectors and define new records according to this ordering. The distributional theory of bivariate records and record times is given. Examples and graphical illustrations are provided. Challenging unsolved problems on record theory of multivariate random sequences are discussed.

Keywords: order statistics, record values, multivariate orderings.

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The role of record values in reliability^{*}

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Abstract. The notion of record values was introduced by Chandler (1952). Since then, a great number of results have been provided for record values. The record values have other interpretations mainly in reliability, such as failures times of minimal repair policies and the relevation transform. Under this interpretations two main areas of research have been developed along the years. One is to study aging properties of record values, such as IFR, NBU or ILR (see Pellerey, Shaked and Zinn, 2000) and the other one is the comparison of record values arising from different parent populations (see Belzunce, Lillo, Ruiz and Shaked, 2001). The purpose of this talk is to provide, first, a historical review of the different interpretations of record values in reliability and a review of some of the main results about aging properties and comparison of record values. Next I will present some new questions about record values, that can be addressed from the point of view of reliability. Mainly I will discuss some new results about the comparison of a minimal repair process with a renewal process and some new results about the role of relevation in allocation of redundant components. This talk is also intended to be a tribute to Moshe Shaked, who made great contributions on the topics of this talk.

Keywords: record values, minimal repair process, relevation transform.

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^{*} This work has been supported by the Ministerio de Economía y Competitividad under grant MTM2012-34023-FEDER

On sufficient conditions for the comparison of hazard rates and mean residual lives based on the likelihood ratio*

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Abstract. The behaviour of the likelihood ratio of two densities f and g has interest in several areas. In fact, in the context of stochastic orders, if the likelihood ratio g/f is increasing, then the corresponding random variables are said to be ordered in the likelihood ratio order, and this property leads to a great number of results in reliability, stochastic processes, risk theory and so on (see Ross, 1996, Müller and Stoyan, 2002 and Shaked and Shanthikumar, 2007 and Belzunce, Martínez-Riquelme and Mulero, 2015). The unimodality of the likelihood ratio is also important. In particular, Metzger and Rüschendorf (1991) proved that if the likelihood ratio g/f is unimodal with mode m , then the ratio of the survival functions $\overline{G}/\overline{F}$ is also unimodal with mode $M \leq m$, where $\overline{F} \equiv 1 - F$, for any distribution function F . However, this result is not true in general. Actually, it may occur that the ratio of the survival functions $\overline{G}/\overline{F}$ is increasing. In this talk we show under which conditions the ratio of the survival functions is unimodal or increasing. These results are then used to provide sets of sufficient conditions for the hazard rate and mean residual life orders.

Keywords: hazard rate, mean residual life, likelihood ratio.

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* This work has been supported by the Ministerio de Economía y Competitividad under grant MTM2012-34023-FEDER

Some new results in ranked set sampling

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Abstract. After McIntyre (1952) introduced RSS to estimate the population mean, many authors have used RSS in the parametric setting (Bhoj and Ahsanullah, 1996; Bhoj, 1997a; Lam, Sinha and Zong, 1994, 1995; Stokes, 1995). One of the popular schemes is to use the median ranked set sampling (MRSS) (see Bhoj 1997a; Muttalak, 1997). Here we propose few different schemes to improve on RSS as well as MRSS. A ranked set sampling procedure with unequal samples and unequal replications (RSSUR) is proposed and used to estimate the population mean. It is shown that the relative precisions of the proposed estimator are higher than the estimators based on RSSU, MRSS and RSS for unimodal symmetric and moderately asymmetrical distributions. Another scheme is proposed for positively skew distributions (RSSUS). It is also compared with the estimators based on (RSS) and (MRSS) procedures. It is observed that the relative precisions of the estimators based on RSSUS are higher than those of the estimators based on RSS and MRSS procedures.

Keywords: heavy right tail distributions, lognormal distribution, mean square error, median ranked set sampling.

Characterizations of continuous distributions by regression of non-adjacent generalized order statistics

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Abstract. Let $X_*^{(r)}, X_*^{(s)}$, $1 \leq r < s$, denote generalized order statistics with fixed parameters $\gamma_1, \dots, \gamma_s$, based on an absolutely continuous distribution function F supported on the interval (α, β) . We consider the problem of unique identification of F by the knowledge of the regression function

$$\xi(x) = E(h(X_*^{(s)}) | X_*^{(r)} = x), \quad x \in (\alpha, \beta),$$

where $h : (\alpha, \beta) \rightarrow \mathbb{R}$ is known continuous and strictly increasing function. Utilizing Markov property of generalized order statistics we give necessary and sufficient condition for the uniqueness of characterization. It provides new almost elementary proof of well-known characterization of exponential power and Pareto distributions by linearity of the regression in the case when $h(x) = x$.

Keywords: regression function, generalized order statistics.

Inference based on sequential order statistics with shape-adjusted hazard rates

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Abstract. For successive failure times of components in a technical system, a flexible model based on sequential order statistics is proposed. Beyond the common assumption of proportionality, it allows for structural adjustments of the hazard rates of the underlying lifetime distributions in situations, where failures have an impact on the entire shape of the hazard rate of remaining components. In the talk, different methods for estimation of the parameters in the model are considered. The talk is based on joint work with Stefan Bedbur, Udo Kamps and Maria Kateri.

Keywords: sequential order statistics, proportional hazard rate, maximum likelihood estimation, Bayes estimation, order restricted inference.

Recurrence relations for the single and product moments of order statistics from the standard two-sided power distribution

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Abstract. The standard two-sided power (STSP) distribution was introduced by Van Dorp and Kotz (2002). The STSP distribution is a beta-like distribution since it is defined on a bounded support and has a similar flexibility. It is especially useful for modeling financial data with excess kurtosis. The distribution has also applications in risk analysis such as PERT (Program Evaluation and Review Technique) (Kotz and Van Dorp, 2004). On the other hand, it is well known (see e.g. David and Nagaraja, 2003) that order statistics and their moments play important roles in many inference problems such as in estimation and developing tests. Especially, single and product moments are useful in deriving BLUE's (Best Linear Unbiased Estimators) in solving location-scale estimating problem of a given data alternatively. Therefore, moment (recurrence) relations are important for computational efficiency since they reduce the amount of calculations required for the evaluations of the moments. In this talk, moments of order statistics from the STSP distribution are first presented (Çetinkaya and Genç, 2015). Since the pdf (probability density function) and the cdf (cumulative distribution function) are piecewise functions, the notion of partial moments occurs with respect to the two pieces of the distribution. Then, the recurrence relations for the single and product partial moments of order statistics from the STSP distribution are given.

Keywords: order statistics, moments, product moments, recurrence relations, the standard two-sided power distribution.

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On the mean estimation in ranked set sampling with combining the information of multiple rankers using fuzzy set approach*

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Abstract. Ranked set sampling (RSS) is an effective method to obtain data for getting information and inference about the population. The main impact of RSS is to use the ranking information of the units in the sampling mechanism. In practice, the ranking process is done without actual measurement and it may cause uncertainty and ranking error. Modeling the uncertainty in the ranking mechanism is a good way of dealing with this ranking error. In literature, there are some studies focused on modeling this uncertainty with probabilistic perspective. In this study, we propose Fuzzy-weighted Ranked Set Sampling (FwRSS) using fuzzy set approach for modeling the uncertainty in ranking process and combining the information coming from multiple rankers. We give a new estimator for population mean and investigate its properties over its competitor estimators. The new sampling method improves the ranking accuracy using fuzzy sets for rank decisions of each ranker and using three different fuzzy norm operators to combine the decisions of all rankers.

Keywords: ranked set sampling, fuzzy sets, mean estimation, multiple rankers.

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* This research was supported by The Scientific and Technological Research Council of Turkey (TUBITAK) under grant 115F300.

A particular signature of repairable coherent systems

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Abstract. We present a particular signature in repairable coherent systems resembling the Samaniego's notion for independent and identically distributed lifetimes. The repairable systems are made of different components which can individually fail, and be minimally repaired up to a fixed number of times. Failures occur according to Poisson processes, which might have either the same intensity function for each component or different ones. The former case reminds the notion of signature presented by Samaniego for i.i.d. random variables, whereas here independent Poisson processes with identical intensity function are considered. An explicit expression for computing the generalized signature of repairable series systems is obtained. It is shown that the reliability function of any repairable coherent system can be expressed as a generalized mixture of the probabilities of the number of repairs until system failure. We also establish that the stochastic ordering between the generalized signatures of two repairable systems is preserved by their lifetimes.

Keywords: Minimal path set, minimal repair, nonhomogeneous Poisson process, signature, stochastic ordering.

Progressive censoring and systems data

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Abstract. In this talk, a new connection between the distribution of component failure times of a coherent system and (adaptive) progressively Type-II censored order statistics is established. Utilizing this property, inferential procedures are developed for failure data of a coherent system when the data is given by all component failures until system failure in two scenarios:

- complete information, i.e., we assume that the failed component is also observed.
- incomplete information, i.e., we have only information about the failure times but not about the components which have failed.

In the first setting, we show that inferential methods for adaptive progressively Type-II censored data can directly be applied to the problem. For incomplete information, we face the problem that the corresponding censoring plan is not observed and that the available inferential procedures depend on the knowledge of the employed censoring plan. In order to get estimates for distributional parameters, we propose maximum likelihood estimators which can be obtained by solving the likelihood equations directly or via an EM-algorithm type procedure. For an exponential distribution, we discuss also a linear estimator to estimate the mean. Moreover, we establish exact distributions for some estimators in the exponential case which can be used, e.g., to construct exact confidence intervals. The results are illustrated by a five component bridge system.

Furthermore, we show that the distribution of a coherent system based on IID components can be written as a mixture of the distributions of progressively Type-II censored order statistics. The coefficients in that representation are called the progressive censoring signature (PC-signature) of the system. We explore the basic properties and potential applications of these mixture representations. Thus, we show that it can be used to study the possible censoring schemes (and their exact probabilities) in the operating development of the system. So it gives different information to that provided by the classical (Samaniego) signature of the system which provides the probability of system failure with the i th component failure. It can also be used to obtain some distribution-free ordering properties for systems.

Keywords: progressive censoring, coherent systems, signature.

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An ergodic theorem for proportions of near order statistics observations

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Abstract. In this talk, we will show that, in the setting of strictly stationary sequences, proportions of observations that fall into a random region determined by a given Borel set and a central order statistic converge almost surely under suitable conditions. We will describe the distribution of the limiting random variable using the concept of conditional quantile. From this general result we will derive a corollary that in the case of strictly stationary and ergodic sequences, these proportions are strongly consistent estimators of certain probabilities, provided that the corresponding population quantile is unique. We will also describe three types of possible asymptotic behavior of these proportions in the case of non-unique population quantile.

Keywords: near order statistic observations, stationary processes, quantiles, conditional quantiles, almost sure convergence.

Asymptotic distributions of exceedance statistics in bivariate random threshold models

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Abstract. Let $Z_1 = \{(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)\}$ be a finite set of random variables with bivariate distribution function $F_{X,Y}$ and $Z_2 = \{(X_{n+1}, Y_{n+1}), (X_{n+2}, Y_{n+2}), \dots, (X_{n+m}, Y_{n+m}), \dots\}$ be a sequence of random vectors with common distribution function $G_{X,Y}$. We assume that Z_1 and Z_2 are independent. Firstly, we consider a simple random threshold model based on number of observations of Z_2 falling into random set $(-\infty, X_1] \times [-\infty, Y_1]$ and derive the finite and asymptotic distribution of exceedance statistic. Secondly we consider a random threshold model based on order statistics and concomitants. More precisely, we investigate the finite and asymptotic distributions of exceedance statistic defined as the number of observations of Z_2 falling into random set $(-\infty, X_{r:n}] \times (-\infty, Y_{[r:n]})$, where $X_{r:n}$ and $Y_{[r:n]}$ are respectively the r th order statistic and its concomitant constructed from the sample Z_1 . For different bivariate distributions the graphs of obtained finite and limiting distributions are provided.

Keywords: threshold models, order statistics, concomitants.

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On the sample median of a jump type distribution on a bounded domain

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Abstract. It is well known that (see e.g. [1]) the median of a sample of an odd size is just the middle order statistic, and that the distribution of it can be easily formulated as long as the pdf (probability density function) and the cdf (cumulative distribution function) of the underlying distribution are represented by simple functions. On the other hand, the median of a sample of an even size is the average of the two middle order statistics, and deriving its distribution requires some calculations. The complexity of the problem is doubled especially for the jump type distributions with bounded supports. In this paper we obtain general results for the pdf and the moments of the median of a sample of even size from a two-piece distribution. We use integral calculus, which was also employed by several authors e.g. [2], [3], [4] for different purposes, to derive the exact pdf and the moments in the case of even sample size. The general results are then used to obtain the required distribution in the case of the two-piece uniform distribution and the two-sided power distribution. Further, the variances of the sample median are calculated for different sample sizes and parameter values.

Keywords: jump type distribution, sample median, special functions.

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B-splines in generalized progressive hybrid censoring

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Abstract. Generalized hybrid censoring schemes as proposed in [1] are extended to progressively Type-II censored data. Using the spacings' based approach due to [2], we obtain explicit expressions for the density functions of the MLEs. The resulting formulas are given in terms of B-spline functions so that they can be easily and efficiently implemented on a computer.

Keywords: maximum likelihood estimation, exponential distribution, generalized type-I progressive hybrid censoring, generalized type-II progressive hybrid censoring, B-spline.

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Upper non-positive bounds on expectations of generalized order statistics from decreasing density populations

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Abstract. We present the upper non-positive bounds on the expectations of gOSs centered about the sample mean, which are based on the parent distributions having decreasing density. Such bounds can be obtained only for particular cases of gOSs and they are expressed in units generated by the central absolute moments of a fixed order. The attainability conditions are also described. The method of deriving presented bounds is based on the maximization of some norms on the properly chosen convex sets. The paper complements the results of Bieniek (2008).

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The inverse Weibull distribution as a failure model under various loss functions and based on progressive first-failure censored data

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Abstract. In this article we consider statistical inferences about the unknown parameters of the inverse Weibull distribution based on progressively first-failure censoring using Bayesian procedures. The Bayes estimators are obtained based on both the symmetric and asymmetric (*Linex*, General Entropy and Precautionary) loss functions. There are no explicit forms for the Bayes estimators; therefore, we propose the Lindley's approximation method to compute the Bayes estimators. A comparison between these estimators and the maximum likelihood estimator (*MLE*) is provided by using extensive simulation and two criteria, namely, the bias and the mean squared error. It is concluded that the approximate Bayes estimators outperform the *MLEs* most of the time. Real life data example is provided to illustrate our proposed estimators.

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Likelihood inference for the component lifetime distribution based on progressively censored systems data

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Abstract. Point and interval estimators for the scale parameter of the component lifetime distribution of a k -component parallel system are obtained when the component lifetimes are assumed to be independently and identically exponentially distributed. The maximum likelihood estimator (MLE) of the scale parameter based on progressively Type-II censored system lifetimes is shown to be unique. It can be obtained by a fixed-point iteration procedure. Exact confidence intervals are constructed by a transformation using normalized spacings. Further, other component lifetime distributions including Weibull distribution are discussed. Finally, an extension to k -out-of- n systems is presented.

Keywords: progressive Type-II censoring, MLE, fixed-point iteration, exact confidence intervals, exponential distribution, Weibull distribution, parallel system.

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Exact inference for the parameter of Lindley distribution under progressive type-I censoring

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Abstract. Lindley distribution (Lindley, 1958) with probability density function

$$f(x; \theta) = \frac{(1+x)e^{-x/\theta}}{\theta(1+\theta)}, \quad x > 0, \theta > 0,$$

can be used as an alternative to the exponential in order to describe a stochastically slightly larger distribution which is possibly unimodal. The distribution is a biased version of exponential (with biasing function $1+x$) and can be represented as a mixture of exponential with mean θ and gamma $\mathcal{G}(2, \theta)$ distributions. Several authors have discussed asymptotic, Bayesian and bootstrap inference for functions of θ based on complete, hybrid and progressively type-II censored samples. However, there is nothing so far on exact inference even in the complete sample case, where the exact distribution of the MLE of θ can be easily derived based on a result of Al-Mutairi et al. (2013). In this talk I will describe exact inference for θ under progressive type-I censoring. I will show that its MLE, $\hat{\theta}$, exists and is unique and I will present its exact distribution. The stochastic monotonicity of $\hat{\theta}$ with respect to θ will be also established and exact confidence intervals will be constructed by pivoting its CDF (when the sample size is small) or by using the method of Bølviken and Skovlund (1996) (when the sample size is moderate).

Keywords: Lindley distribution, progressive type-I censoring, maximum likelihood estimation, exact inference.

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The rank of bivariate extreme order statistics: An application in hydrology

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Abstract. Hydrological events can be described as a multivariate event whose main characteristics can be summarized by dependent random variables. A flood event is one of the complex hydrological event which can be characterized by its peak, volume and duration, which are mutually dependent. For multivariate flood frequency analysis, the joint probabilistic behaviour of two or more correlated random variables which characterize the event should be investigated. Many researchers have showed that a bivariate gamma distribution with special gamma marginals may be useful for representing the joint behaviour of multivariate hydrological events such as floods and storms. For more details, one can see Blokhinov and Sarmanov (1968), Clarke (1980) and Yue et al (2001). In the paper of Yue et al (2001), a few bivariate Gamma distribution models which are Izawa, Moran, Smith-Adelfang-Tubbs (SAT) and Farlie-Gumbel-Morgenstern (FGM) constructed from specified Gamma marginals are summarized and the applicability of these models have been showed with actual flood data. In this study, the joint distribution of the rank of bivariate order statistics is obtained by using the bivariate binomial distribution. The obtained joint distribution of rank of bivariate extreme order statistics can be applied in analyzing and investigating the joint probabilistic behaviour of environmental events such as floods and storms. By using the same data obtained from the paper Yue et al (2001), we represent the joint probabilistic behaviour of largest flood peak and largest flood volume using the obtained joint distribution of rank of bivariate extreme order statistics in a new sample. The basis of this joint probability is important to identify the estimate of flood risk created by extreme events.

Keywords: bivariate order statistics, extreme value, hydrology, risk.

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A new bivariate semiparametric control chart based on order statistics and concomitants

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Abstract. One of the traditional techniques for the construction of nonparametric control charts involves the use of two independent samples, the reference and the test sample. Data collected from the former sample are exploited for setting up the decision rule. Subsequently, successive test samples are drawn and the decision rule is applied in order to ascertain whether the underlying process has shifted to an out-of-control distribution or not.

In the present work, we introduce a new semiparametric (one-sided) control chart which is based on the bivariate statistic $(X_{r:n}, Y_{s:n})$, where $X_{r:n}$ and $Y_{s:n}$ are the order statistics of the respective X and Y test samples. For our decision making, the test statistic is compared to the values of the specific order statistics and concomitants of the reference sample.

A key advantage of the chart, beside its simplicity, is the fact that the FAR and ARL_{in} values are not affected by the marginal distributions of the monitored characteristic, and, at the same time, they do not change dramatically when different copulas are used. As a result, it behaves almost as a purely nonparametric control chart, although it is typically affected by the dependence structure of the monitored characteristics.

An explicit expression for the operating characteristic function of the new chart is obtained. In addition, tables are provided for the implementation of the suggested control scheme for some typical Average Run Length values and (False) Alarm Rates. Finally, its performance is contrasted to that of alternative nonparametric control charts that have already appeared in the related literature.

Keywords: order statistics, concomitants, statistical quality control, copulas.

A modified Wilcoxon-type rank-sum control chart

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Abstract. Control charts help the practitioners to identify assignable causes so that the state of statistical control can be achieved. In the event of having an undesirable shift in the process, a control chart is expected to detect it as quickly as possible and give an out-of-control signal. In this article, we introduce a new Wilcoxon-type rank-sum control chart which, due to its nature, does not require the assumption of any specific probability distribution for the underlying process (measurements). The proposed chart combines the idea of precedence testing and Wilcoxon rank-sum statistic. More specifically, while the process is in-control, a reference sample is drawn from it and after completing Phase I, test samples are drawn independently of each other (and also of the reference sample). The decision whether the observed process is still in-control or not, is based on the sum of ranks of those observations of the test sample that are situated between the a -th and the b -th order statistic of the reference sample. The suggested chart is quite useful when data are not completely observed. Tables are provided for the implementation of the proposed chart for some typical false alarm rates and a numerical comparison to classical Shewhart-type or alternative nonparametric charts is also presented.

Keywords: distribution-free control charts, precedence tests, Lehmann alternatives, Shewhart-type chart, statistical process control.

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**Sharp bounds on the expectations
of linear combinations of k th records
expressed in the Gini mean difference units**

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Abstract. We describe a method of calculating sharp lower and upper bounds on the expectations of linear combinations of k th records expressed in the Gini mean difference units of the original i.i.d. observations. In particular, we provide sharp lower and upper bounds on the expectations of k th records and their differences. We also present the families of distributions which attain the bounds in the limit.

Keywords: expectation, Gini mean difference, sharp bound, k th record.

Ascents of random sequences

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Abstract. Let $\{X_i\}_{i \geq 1}$ be a sequence of iid random variables from an absolutely continuous distribution. We say that there is an ascent at position i if $X_i > X_{i-1}$, $i \geq 2$, then the random variable X_i is called an ascent-value and i the corresponding ascent-position. In the present work we investigate the distribution theory of the sequence $\{A_k\}_{k \geq 1}$, where A_k denotes the k -th ascent-value of the sequence $\{X_i\}_{i \geq 1}$. We obtain the joint and marginal distributions of ascent-values and ascent-positions, moments and asymptotic properties of these statistics. We also study the connection with the ascents and descending runs (runs-down) of random permutations.

Keywords: ascents, descents, runs, random permutations, Markov chains, compact operators, asymptotic properties.

Bounds on reliability of coherent systems with heterogeneous components

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Abstract. We consider coherent systems with heterogeneous components having life-time dependence structure given by a survival copula. We present sharp bounds on system reliability in the unit of arithmetic mean of element reliabilities. The bounds do not depend on the copula, they are simple functions of the system structure. The proofs are based on computations on k -out-of- n systems. As a corollary, bounds on system expected lifetime are provided.

Keywords: coherent system, reliability, bounds, dependent components, k -out-of- n system, expected lifetime.

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A very short proof of the multivariate Chebyshev's inequality. Applications to order statistics and data sets

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Abstract. We will see a very simple proof of the Chebyshev's inequality for random vectors. This inequality provides a lower bound for the percentage of the population of an arbitrary random vector $\mathbf{X}$ with finite mean $\mu = E(\mathbf{X})$ and a positive definite covariance matrix $V = Cov(\mathbf{X})$ whose Mahalanobis distance with respect to V to the mean μ is less than a fixed value. We will see that this bound is sharp (i.e. it is the best possible bound when we just know μ and V). An alternative bound will be provided for singular matrices V by using the principal components. We will apply this inequality to order statistics and general data sets. The results included in this talk were obtained in the references given below.

Keywords: Chebyshev (Tchebychev) inequality, Mahalanobis distance, principal components, concentration ellipsoid, order statistics.

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Bounds for distribution functions of order statistics from a sample of dependent observations with known multidimensional marginal distributions

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Abstract. Let $X = (X_1, \dots, X_n)$ be an n -tuple of random variables X_i each having the same known distribution function F and such that each k -tuple has the same known copula. We extend the Kemperman (1997, Bounding moments of an order statistic when each k -tuple is independent. In: *Distributions with given marginals and moment problems*, pp. 291-304, ed. by Victor Beneš and Josef Štěpán) characterization and resulting method for determining sharp lower and upper bounds for the distribution functions of the associated order statistics to the case of: (i) dependence structures other than independence, (ii) arbitrary linear combinations of distribution functions of order statistics.

Keywords: dependent random variables, order statistics, distribution bounds.

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Statistical inference based on judgment post-stratified samples in finite populations

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Abstract. This paper draws statistical inference for finite population mean based on judgment post stratified (JPS) samples. The JPS sample first selects a simple random sample and then stratifies the selected units into H judgment classes based on their relative positions (ranks) in a small set of size H . This leads to a sample with random sample sizes in judgment classes. Ranking process can be performed either using auxiliary variables or visual inspection to identify the ranks of the measured observations. The paper develops unbiased estimator and constructs confidence interval for population mean. Since judgment ranks are random variables, by conditioning on the measured observations we construct Rao-Blackwellized estimators for the population mean. The paper shows that Rao-Blackwellized estimators perform better than usual JPS estimators. The proposed estimators are applied to 2012 United States Department of Agriculture Census Data.

Keywords: post stratified sample, finite sample correction, ranked set sample, stratified sample, Rao-Blackwellized estimator.

On a discrete analogue of Terrell's characterization of rectangular distributions

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Abstract. Let X_1 and X_2 be two i.i.d. r.v.'s with finite non-zero variance. Terrell (1983) proved that the Pearson correlation coefficient of $X_{(1)} = \min\{X_1, X_2\}$ and $X_{(2)} = \max\{X_1, X_2\}$, $\rho(X_{(1)}, X_{(2)})$, is no greater than $1/2$, and attains its maximal value if and only if the parent distribution is rectangular (uniform over some interval). This result was extended by Székely and Móri (1985) to order statistics $X_{r:n}$ based on a sample of size n , namely, they showed that

$$\rho(X_{i:n}, X_{j:n}) \leq \sqrt{\frac{i(n+1-j)}{j(n+1-i)}}, \quad 1 \leq i < j \leq n,$$

with equality only for rectangular distributions. Subsequently, López-Blázquez and Castaño-Martínez (2006) showed that the same inequality holds for the order statistics based on a without-replacement sample from a finite population. Moreover, a general method for deriving this kind of results was provided by Papadatos and Xifara (2013); it should be noted that all the above results are closely connected to the notion of maximal correlation coefficient.

In the present talk we investigate a discrete analogue of Terrell's characterization. A natural question is as follows: Suppose that the i.i.d. r.v.'s X_1, X_2 assume the values $x_1 < x_2 < \dots < x_N$, each with probability $1/N$. What is the maximal value of $\rho(X_{(1)}, X_{(2)})$, and what kind of distributions on the N points attain the maximal value? Our investigation provides the following answer:

$$\rho(X_{(1)}, X_{(2)}) \leq \frac{1 - 1/N^2}{2 + 1/N^2},$$

with equality if and only if $x_j = c + j\lambda$, $j = 1, 2, \dots, N$, for some $c \in \mathbb{R}$ and $\lambda > 0$. [We note that our proof uses Hahn polynomials, and is not yet complete; however, a rigorous proof is available for $N \leq 18$.]

Keywords: Terrell's characterization, order statistics, maximal correlation, discrete distributions.

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On the general class of flexible Weibull distributions

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Abstract. We consider a general class of modified Weibull distributions, generated by a combination of two cumulative hazard functions. Some well-known modified Weibull distributions, such as modified Weibull distribution (Lai *et al.*, 2003) and flexible Weibull (Bebbington *et al.*, 2007), belong to the class. We propose a new modified Weibull distribution which also belongs to the class, and show that its hazard function can be monotone, bathtub-shaped, modified bathtub-shaped, or even upside-down bathtub-shaped. We also discuss the estimation method for its parameters such as least square estimation and maximum likelihood estimation. Using examples, we compare the suggested distribution with some well-known modified Weibull distributions, and show that the suggested distribution shows good performances.

Keywords: bathtub shape, goodness of fit, hazard function, maximum likelihood estimate, reliability.

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Induced generalized record ranked set sampling

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Abstract. In this work we consider a situation where the variable Y of primary interest on the units of an infinite population is expensive for its measurement while making measurement of an auxiliary variable X which is jointly distributed with Y is very easy and inexpensive so that it can be measured from any number of units. It is well-known that classical record values of a sequence of observations terminates without producing any further records when the scrutiny of the observations reaches at an outlier in the data. However the generalized (k) record values measured on the units for the auxiliary variable X has an inbuilt system to escape unhurt from the effect of $k - 1$ outliers in the sequence of units. Thus if we rank the units belonging to each of the n sets used for any RSS based on the generalized (k) record values observed on the auxiliary variable X and make measurement with respect to the variable Y of primary interest on the selected units only, then the resulting ranked set sampling is known as “Induced Generalized Record Ranked Set Sampling (IGRRSS)”. We propose estimators of the parameters associated with the variable Y of primary interest based on observations of the proposed IGRRSS when the parent population follows a bivariate normal distribution. A primary data collected by IGRRSS method is demonstrated and the obtained data used to illustrate the results developed in this work.

A generalized cumulative residual entropy with applications

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Abstract. Recently, Psarrakos and Navarro (2013) proposed a measure of uncertainty which extends the cumulative residual entropy, called the generalized cumulative residual entropy (GCRE). The new entropy is related to the record values of a sequence of i.i.d. random variables. In the present talk, properties and applications of the GCRE are explored. Bounds, stochastic order properties and characterization results are also discussed. The GCRE is studied as a risk measure and is compared to the standard deviation and the right-tail risk measure. Furthermore, a sequence of weighted distributions is introduced based on the GCRE. This sequence includes the mean residual weighted distribution. Discrete mixtures among the new sequence of weighted distributions and the length-biased distribution are also studied. Numerical examples are given to illustrate our results.

Keywords: generalized cumulative residual entropy, mean residual lifetime, hazard rate, characterization results, weighted distributions, stochastic order, risk measures, record values.

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A family of premium principles based on mixtures of TVaRs

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Abstract. Risk-adjusted distributions are commonly used in actuarial science to define premium principles. In this paper, we claim that an appropriate risk-adjusted distribution, besides of satisfying other desirable properties, should be well-behaved under conditioning with respect to the original risk distribution. Based on a sequence of such risk-adjusted distributions, we introduce a family of premium principles that gradually incorporate the degree of risk-aversion of the insurer in the safety loading. Members of this family are particular distortion premium principles than can be represented as mixtures of TVaRs, where the weights in the mixture reflect the attitude toward risk of the insurer. We make a systematic study of this family of premium principles.

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On generating bivariate distributions using the distributions of concomitants of extreme order statistics

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Abstract. In this talk the main interest is to expose the importance of knowledge on the distribution of concomitants of either smallest order statistic or largest order statistic and the marginal distribution on the other variable in the identification of the appropriate parent bivariate distribution. We further describe about a systematic mathematical approach which helps in the identification of the parent bivariate distribution. Using the above theory we illustrate how new bivariate models can be generated by assuming specified forms for the distributions of concomitants of extreme order statistics. We describe the applications of some such generated models in modeling using bivariate data set arising from a population.

Stochastic order comparisons between extreme order statistics in the last decades

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Abstract. In recent years, stochastic ordering relations between extreme order statistics from parametric families of distributions have been studied extensively by many researchers. Results for heterogeneous exponential distributions can be found, for instance, in [10], [15] and [14]; for heterogeneous gamma distributions in [16], [9] and [17]; for heterogeneous Weibull distributions in [4], [3] and [12], among others. There are, of course, earlier works in the literature. Please refer to a review article [7] and references therein for more details.

It is of natural interest to consider other families of random variables. Many interesting results have been obtained in the literature for general models such as proportional hazard rate (PHR), proportional reversed hazard rate (PRHR) and scale models. [6] and [1] investigated stochastic comparisons for PHR models. The PRHR model was introduced as a dual of the well known PHR model. Some recent articles on these subjects are, e.g., [2] and [13]. Recent results on the scale model are given in [5], [11] and [8].

In this talk, we review the remarkable progress done over the last few decades at comparing extreme order statistics according to different stochastic orders.

Keywords: stochastic orderings, order statistics, parallel systems, series systems.

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Inequalities for order statistics from distributions with monotone hazard rate

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Abstract. In this work, we present some new inequalities for the expectation of order statistics of samples from distributions with monotone hazards rate. The inequalities are used to obtain new characterizations of the exponential distribution. Let $X_1, \dots, X_n$ be independent, identically distributed, nonnegative, absolutely continuous random variables with distribution function $F(x)$. Denote the respective order statistics by $X_{(1)}, \dots, X_{(n)}$. One of the main results is given by the following

Theorem. If $F(x)$ has increasing hazard rate, then

$$\begin{aligned} \left(\frac{1}{n} + \frac{1}{n-1} + \dots + 1\right)^{-1} EX_{(n)} &< \left(\frac{1}{n} + \frac{1}{n-1} + \dots + \frac{1}{2}\right)^{-1} EX_{(n-1)} < \\ &< \dots < \left(\frac{1}{n}\right)^{-1} EX_{(1)}. \end{aligned}$$

If $F(x)$ has decreasing hazard rate, then

$$\begin{aligned} \left(\frac{1}{n} + \frac{1}{n-1} + \dots + 1\right)^{-1} EX_{(n)} &> \left(\frac{1}{n} + \frac{1}{n-1} + \dots + \frac{1}{2}\right)^{-1} EX_{(n-1)} > \\ &> \dots > \left(\frac{1}{n}\right)^{-1} EX_{(1)}. \end{aligned}$$

Keywords: inequalities, characterization, monotone hazard rate, exponential distribution, order statistics.

Performance comparison of quality control charts based on ranked set sampling

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Abstract. In this study, the performance of some quality control charts based on different Ranked Set Sampling (RSS) schemes will be compared. The average run length (ARL) of the charts are going to be calculated for different distributions using various sample sizes with the aim of examining their performances.

Keywords: ranked set sampling, quality control charts, order statistics, average run length.

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New characterizations of exponential distribution

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Abstract. Characterizations of the exponential distribution are obtained based on equality in distribution involving order statistics. In particular, it is proven that the well-known Sukhatme-Rényi necessary condition is also sufficient for exponentiality. A new technique of proof due to Arnold and Villaseñor for samples of size two is adapted to deal with the general case of samples of size n for any fixed $n \geq 2$. We argue that this new method, making use of the Maclaurin series expansion of the exponential density, has the potential of proving more characterization results.

Keywords: characterizations, exponential distribution, order statistics, random translations.

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